

Physics 222 (fall 96): Solutions for Homework Set #1

19/Q4 Is it possible to orient a current loop in a uniform magnetic field so that the loop will not tend to rotate? Explain.

Answer: This question becomes trivial with the results of Serway, section 19.4 (which we have not yet discussed in class). However, let's try to answer this question using 19.3 only. Once we are successful, the concepts in 19.4 will make much more sense. Here we go:

For simplicity, let's assume that the loop has the form of a square, that the current I flows clockwise, and that the magnetic field \vec{B} points up. If we align the loop in such a way that its left and right sides are parallel to the magnetic field, the magnetic force \vec{F} acting on these two sides is zero, see Serway Eq. (19.3): $\vec{F} = I\vec{l} \times \vec{B}$. The force on the top side, however, points out of the page and the force on the bottom side points into the page. Therefore, the torque is not zero, i.e., the loop will try to turn. The total force acting on the loop, however, is zero, since the two forces cancel.

Now let's assume that the loop is in the plane of the paper and that the magnetic field points out of the page. If the current still flows clockwise, then the force on the top side points down and is canceled by the force on the bottom side. The force on the right side points to the left and is canceled by the force acting on the left side, which points to the right. In this case, we see that the total force and the torque are zero.

How do we generalize this result: Let's define the magnetic moment $\vec{\mu}$ to be a vector, whose magnitude is the enclosed area A multiplied by the current I in the loop. Its direction is along the surface normal \vec{A} (the direction perpendicular to the plane of the loop) and by the right-hand rule (see Fig. 19.11 in Serway), i.e., $\vec{\mu} = I\vec{A}$. Given the two examples above, we may speculate that the torque is zero, if the magnetic moment is parallel to the magnetic field. More about this in lecture 2.

19.7 PROBLEM: A cosmic
energy of 10 MeV
equal to that of
What is the galactic

SOLUTION: Think

- 19/P14 A wire with a linear mass density of 0.5 g/cm carries a 2 A current horizontally to the south. What are the direction and magnitude of the minimum magnetic field needed to lift this wire vertically upward?

Solution: Three-dimensional problems are difficult, since we can't draw 3-D pictures. They are even more difficult to explain without a picture. Let's assume for the moment that you are trying to solve this problem on a piece of paper that is placed on a flat horizontal surface. Let's draw the four geographic directions on the paper. Just like on a map, let's assume that north is on the top of the page and south on the bottom of the page. East is on the right and west is on the left. Vertically upward means out of the plane of the paper (towards the sky), and vertically downward is into the paper. (Your instructor in recitation may set up this problem differently, if he/she does it on a vertical blackboard.) Therefore, the current I flows from the top of the page to the bottom of the page.

Let's first find the direction of the required magnetic field. Remember that $\vec{F} = I\vec{l} \times \vec{B}$. Since \vec{l} points south and the force \vec{F} is out of the page, the required field \vec{B} points to the right, i.e., eastward. (Use the right-hand rule, Fig. 19.4.)

We find the magnitude of the field required by setting the gravitational force $m\vec{g}$ (weight of the wire) equal to the magnetic force $I\vec{l} \times \vec{B}$. Since the directions of all vectors are perpendicular to each other, we don't have to worry about the vector product. By dividing by the length l of the wire, we find

$$\frac{mg}{l} = IB. \quad \text{Therefore} \quad B = \frac{m}{l} \times \frac{g}{I} = \frac{0.5 \times 10^{-3} \text{ kg}}{10^{-2} \text{ m}} \times \frac{9.81 \text{ m}}{\text{s}^2} \times \frac{1}{2 \text{ A}} = 0.25 \text{ T}.$$

Remember to convert into international units: cm needs to be converted to m and g to kg. Then, the correct international unit for the magnetic field (T or Tesla) will come out. Don't be confused by the term linear mass density. This is simply the mass of the wire divided by its length, i.e., m/l .

- 19/P17 A current of $I=15$ A is directed along the positive x axis in a wire perpendicular to a magnetic field. The wire experiences a magnetic force per unit length of 0.63 N/m in the negative y direction. Calculate the magnitude and direction of the magnetic field in the region through which the current passes.

Solution: This problem is very similar to the previous one. Again, let us first find the direction of the magnetic field. Draw another picture, where the positive x axis points left and the positive y axis points up. In a right-handed coordinate system with this convention, the positive z axis points out of the page. Then, the current I flows to the right and force \vec{F} points down. Using the right-hand rule (Fig. 19.4), we see that the magnetic field \vec{B} points out of the page (along the positive z axis).

The magnitude of the field is

$$B = \frac{F}{l} \times \frac{1}{I} = \frac{0.63 \text{ N/m}}{15 \text{ A}} = 0.042 \text{ T}.$$

19.19 PROBLEM: A strong magnetic field is placed under a horizontal conducting ring of radius a that carries current I as shown in Figure 19.19. Find the magnetic lines of force at the ring's location and the resultant force on the ring.

SOLUTION: The magnetic field is $B = B_0 \hat{z}$. The force on a small element ds of the ring is $d\mathbf{F} = I ds \hat{s} \times \mathbf{B} = I ds B \hat{s} \times \hat{z}$. The resultant force is $\mathbf{F} = \oint d\mathbf{F} = I B \oint ds \hat{s} \times \hat{z}$. The vector \hat{s} is the unit vector in the radial direction, and $\hat{s} \times \hat{z} = \hat{\phi}$, the unit vector in the azimuthal direction. The integral $\oint ds \hat{\phi}$ is zero, so the resultant force is zero.